Lesson 7.4  Identifying Volumes of Composite Solids

For this practice, you may use a calculator. Use 3.14 as an approximation for \( \pi \). Round your answer to the nearest tenth if necessary.

1. Find the volume of each of the following composite solids.
   a) A cylinder that sits on top of a triangular prism
   b) A cone that sits on top of a cylinder
2. A cylinder has a hemispherical hole cut from the top of it. The shaded rim is 3 inches thick. Find the volume of the solid.

[Diagram of a cylinder with a hemispherical hole and dimensions labeled: 10 in. radius, 3 in. thickness, 12 in. height]

3. The diagram shows a composite solid made up of a hemisphere on top of a cone.
   
   a) Find the height of the cone.

   [Diagram of a cone with dimensions labeled: 20 cm radius, 26 cm height]

   b) Find the total volume of the solid.
4. Joanne made a jewelry box in the shape of a rectangular prism with the shape of a square pyramid.

   a) Find the length of a diagonal of the base of the jewelry box.

   b) Find the height of the cover.

   c) Find the total volume of the jewelry box.
5. A metallic cylinder has a cone-shaped hole cut out of it, as shown.
   a) Find the depth of the hole.
   b) Find the volume of the remaining metal.

6. Miley is decorating the Young Writer’s Corner in her class. She makes a large model of a pencil by using a cylindrical piece of wood and a cone at one end for the tip. She uses a hemispherical piece of foam for the eraser at the other end of the pencil.
   a) Find the radius of the eraser.
   b) Find the volume of the entire model of the pencil.
A sculpture is made out of a triangular prism with a pyramid extending out from one of its sides. The sculptor made the length of all the edges of the solid 10 inches.

a) Find the height of the pyramid.

b) Find the volume of the solid.

Raven ordered a new wooden nameplate for her office desk. The nameplate is made of a triangular prism attached to a 0.6 inch high base. The base is in the shape of a rectangular prism. Find the volume of the entire wooden nameplate.
11. **a)** Let the length of the diagonal of the base be \( x \) meters.

\[
\left(\frac{x}{2}\right)^2 + 4.5^2 = 5^2
\]

\[
\left(\frac{x}{2}\right)^2 + 20.25 = 25
\]

\[
\left(\frac{x}{2}\right)^2 + 20.25 - 20.25 = 25 - 20.25
\]

\[
\left(\frac{x}{2}\right)^2 = 4.75
\]

\[
\frac{x}{2} = \sqrt{4.75}
\]

\[
\frac{x}{2} = 2.179
\]

\[
2 \cdot \frac{x}{2} = 2 \cdot 2.179
\]

\[
x = 4.358
\]

The length of the diagonal of the base is 4.358 meters.

**b)** Let the side of the square base be \( y \) meters.

So, the area of the square is \( y^2 \) square meters.

\[
y^2 + y^2 = x^2
\]

\[
2y^2 = 4.538^2
\]

\[
2y^2 = 18.99
\]

\[
y^2 = 9.5
\]

The area of the base of the pyramid is approximately 9.5 square meters.

### Lesson 7.4

1. **a)** Let the unknown side of the base of the triangular prism be \( x \) inches.

\[
x^2 + 20^2 = 25^2
\]

\[
x^2 + 400 = 625
\]

\[
x^2 + 400 - 400 = 625 - 400
\]

\[
x^2 = 225
\]

\[
x = \sqrt{225}
\]

\[
x = 15
\]

Volume of prism

\[
\frac{1}{2} \cdot 15 \cdot 20 \cdot 14
\]

\[
= 600 \text{ in}^3
\]

Volume of cylinder

\[
\approx 3.14 \cdot 3.5^2 \cdot 14
\]

\[
\approx 538.5 \text{ in}^3
\]

Volume of composite solid

\[
= 538.5 + 600
\]

\[
= 1,138.5 \text{ in}^3
\]

The volume of the composite solid is approximately 1,138.5 cubic inches.

2. Let the radius of the hemisphere be \( x \) inches.

\[
x^2 + x^2 = 10^2
\]

\[
2x^2 = 100
\]

\[
x^2 = 50
\]

\[
x = \sqrt{50}
\]

\[
x = 7.07
\]

Volume of hemisphere

\[
\frac{1}{2} \cdot \frac{4}{3} \cdot 3.14 \cdot 7.07^3
\]

\[
= 739.77 \text{ in}^3
\]

Radius of cylinder

\[
= 7.07 + 3
\]

\[
= 10.07 \text{ in.}
\]

Volume of cylinder

\[
\approx 3.14 \cdot 10.07^2 \cdot 12
\]

\[
\approx 3,820.94 \text{ in}^3
\]

Volume of solid

\[
= 3,820.94 - 739.77
\]

\[
= 3,081.2 \text{ in}^3
\]

The volume of the solid is approximately 3,081.2 cubic inches.

3. Radius of cone = 10 cm

**a)** Let the height of the cone be \( x \) centimeters.

\[
26^2 = 10^2 + x^2
\]

\[
676 = 100 + x^2
\]

\[
676 - 100 = 100 + x^2 - 100
\]

\[
x^2 = 576
\]

\[
x = \sqrt{576}
\]

\[
x = 24
\]

The height of the cone is 24 centimeters.
b) Volume of cone $= \frac{1}{3} \cdot 3.14 \cdot 10^2 \cdot 24$
   $= 2,512 \text{ cm}^3$

Volume of hemisphere $= \frac{1}{2} \cdot \frac{4}{3} \cdot 3.14 \cdot 10^3$
   $= 2,093.3 \text{ cm}^3$

Volume of solid $= 2,093.3 + 2,512$
   $= 4,605.3 \text{ cm}^3$

The volume of the solid is approximately 4,605.3 cubic centimeters.

4. a) Let the length of the diagonal be $x$ inches.
   $x^2 = 12^2 + 12^2$
   $= 288$
   $x = \sqrt{288}$
   $x = 17.0$

The length of the diagonal of the base is approximately 17.0 inches.

b) Length of half the diagonal $= \frac{1}{2} \cdot \sqrt{288}$
   $= 8.485$ in.

Let the height of the pyramid be $y$ inches.
   $y^2 + 8.485^2 = 10^2$
   $y^2 + 8.485^2 - 8.485^2 = 10^2 - 8.485^2$
   $y^2 = 28.00$
   $y = \sqrt{28}$
   $y = 5.3$

The height of the pyramid is approximately 5.3 inches.

c) Volume of pyramid $= \frac{1}{3} \cdot 12 \cdot 12 \cdot \sqrt{28}$
   $= 254.0 \text{ in}^3$

Volume of rectangular block $= 12 \cdot 12 \cdot 8$
   $= 1,152 \text{ in}^3$

Total volume $= 1,152 + 254.0$
   $= 1,406.0 \text{ in}^3$

The volume of the solid is approximately 1,406 cubic inches.

5. a) Radius of cone $= 6$ cm

Let the depth of the hole be $x$ centimeters.
   $x^2 + 6^2 = 10^2$
   $x^2 + 36 = 100$
   $x^2 + 36 - 36 = 100 - 36$
   $x^2 = 64$
   $x = \sqrt{64}$
   $x = 8$

The depth of the cone is 8 centimeters.

b) Volume of cylinder $= 3.14 \cdot 6^2 \cdot 18$
   $= 2,034.72 \text{ cm}^3$

Volume of cone-shaped hole $= \frac{1}{3} \cdot 3.14 \cdot 6^2 \cdot 8$
   $= 301.44 \text{ cm}^3$

Volume of remaining metal $= 2,034.72 - 301.44$
   $= 1,733.28 \text{ cm}^3$

The volume of the remaining metal is approximately 1,733.28 cubic centimeters.

6. a) Let the diameter of the base of the cone be $x$ inches.
   $x^2 = 5^2 + 5^2$
   $x = \sqrt{50}$
   $x = 7.071$

Radius of hemisphere
   $= $ Radius of base of cone
   $= \frac{1}{2} \cdot 7.071$
   $= 3.5355$ in.

The radius of the hemisphere is approximately 3.5 inches.

b) Let the height of the cone be $y$ inches.
   $y^2 + 3.5355^2 = 5^2$
   $y^2 + 3.5355^2 - 3.5355^2 = 5^2 - 3.5355^2$
   $y^2 = 12.50$
   $y = \sqrt{12.50}$
   $y = 3.536$

Volume of cone $= \frac{1}{3} \cdot 3.14 \cdot 3.5355^2 \cdot 3.536$
   $= 46.26 \text{ in}^3$

Volume of cylinder $= 3.14 \cdot 3.5355^2 \cdot 25$
   $= 981.23 \text{ in}^3$

Volume of hemisphere
   $= \frac{1}{2} \cdot \frac{4}{3} \cdot 3.14 \cdot 3.5355^3$
   $= 92.51 \text{ in}^3$

Volume of model $= 46.26 + 981.23 + 92.51$
   $= 1,120.0 \text{ in}^3$

The volume of the solid is approximately 1,120 cubic inches.
7. a) Let the diagonal of the pyramid base be \( x \) inches.
\[
\begin{align*}
x^2 &= 10^2 + 10^2 \\
x^2 &= 200 \\
x &= \sqrt{200} \\
x &\approx 14.14
\end{align*}
\]
Length of half the diagonal = \( \frac{1}{2} \cdot 14.14 = 7.07 \) in.

Let the height of the pyramid be \( y \) inches.
\[
\begin{align*}
y^2 + 7.07^2 &= 10^2 \\
y^2 + 7.07^2 - 7.07^2 &= 10^2 - 7.07^2 \\
y^2 &= 50.02 \\
y &= \sqrt{50.02} \\
y &\approx 7.1
\end{align*}
\]
The height of the pyramid is approximately 7.1 inches.

b) Volume of pyramid = \( \frac{1}{3} \cdot 10^2 \cdot \sqrt{50.02} \)
\[
= 235.75 \text{ in}^3
\]

Let the height of the base of the prism be \( z \) inches.
\[
\begin{align*}
z^2 &= 10^2 - 5^2 \\
z^2 &= 100 - 25 \\
z^2 &= 75 \\
z &= \sqrt{75} \\
z &\approx 8.66
\end{align*}
\]
Volume of prism = \( \frac{1}{2} \cdot 10 \cdot 8.66 \cdot 10 \)
\[
= 433 \text{ in}^3
\]
Volume of solid = 433 + 235.75
\[
= 668.75 \text{ in}^3
\]
The volume of the solid is approximately 668.8 cubic inches.

8. Let the height of the triangular face of the triangular prism be \( x \) inches.
\[
\begin{align*}
x^2 + 0.6^2 &= 1^2 \\
x^2 + 0.36 &= 1 \\
x^2 + 0.36 - 0.36 &= 1 - 0.36 \\
x^2 &= 0.64 \\
x &= \sqrt{0.64} \\
x &= 0.8
\end{align*}
\]
Volume of triangular prism
\[
= \frac{1}{2} \cdot 1.2 \cdot 0.8 \cdot 8 \\
= 3.84 \text{ in}^3
\]
Volume of rectangular prism
\[
= 8 \cdot 1.2 \cdot 0.6 \\
= 5.76 \text{ in}^3
\]
Volume of solid
\[
= 3.84 + 5.76 \\
= 9.6 \text{ in}^3
\]
The volume of the entire wooden nameplate is 9.6 cubic inches.

Brain@Work

1. a) Distance from \( C \) to \( P \)
\[
= \sqrt{8 - 4^2 + -10 - 2^2} \\
= \sqrt{4^2 + 12^2} \\
= \sqrt{160} \text{ units}
\]
Distance from \( C \) to \( Q \)
\[
= \sqrt{6 - 4^2 + 8 - 2^2} \\
= \sqrt{2^2 + 6^2} \\
= \sqrt{40} \text{ units}
\]
Distance from \( C \) to \( R \)
\[
= \sqrt{4 + 8^2 + 2 - 6^2} \\
= \sqrt{12^2 + 4^2} \\
= \sqrt{160} \text{ units}
\]
Distance from \( C \) to \( S \)
\[
= \sqrt{4 + 8^2 + 2 + 2^2} \\
= \sqrt{12^2 + 4^2} \\
= \sqrt{160} \text{ units}
\]

Three points \( P \ (8, -10), \ R \ (-8, 6), \) and \( S \ (-8, -2) \) are the same distance from point \( C \). The distance of point \( Q \ (6, 8) \) from point \( C \) is not the same as the rest. Therefore, point \( Q \) is not on the circle.

b) The radius of the circle is \( \sqrt{160} \text{ units} \).
Since point \( Q \) is at a distance of \( \sqrt{40} \text{ units} \) from point \( C \) which is less than the radius \( \sqrt{160} \text{ units} \), point \( Q \) lies inside the circle.

2. Surface area of bigger cylinder
\[
\approx 2 \cdot 3.14 \cdot 5 \cdot 6 + 3.14 \cdot 5^2 \\
= 188.4 + 78.5 \\
= 266.9 \text{ in}^2
\]
Curved surface area of smaller cylinder
\[
\approx 2 \cdot 3.14 \cdot 3 \cdot 6 \\
= 113.04 \text{ in}^2
\]
Height of entire cone
\[
= 4.5 + 6.5 \\
= 11 \text{ in.}
\]
Let the slant height of the entire cone be \( x \) inches.
\[
\begin{align*}
y^2 + 5^2 &= 11^2 \\
y^2 + 25 &= 121 \\
y^2 &= 96 \\
y &= \sqrt{96} \\
y &\approx 9.80
\end{align*}
\]
Curved surface area of entire cone
\[
\approx 3.14 \cdot 5 \cdot 9.80 \\
= 153.86 \text{ in}^2
\]